Sequences, Series, and Recursion

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Problem Section 1

1. Above, we saw that n^2, a quadratic, has a linear finite difference sequence. Prove this for all quadratic sequences.
2. What about cubics?
3. What about an nth order polynomial?

Problem Section 2

1. The "Tribonacci" sequence is defined by $T_{n+3} = T_{n+2} + T_{n+1} + T_n$ and the starting values $T_1 = T_2 = T_3 = 1$. Find the smallest n for which T_n is over 9000. A computer would be helpful, but don't just brute force it.
2. Given a set of 3 initial values, what does the sequence $a_{n+3} = 3a_{n+2} - 3a_{n+1} + a_n$ do?

Problem Section 3

1. Find a recursive definition for the sequence whose closed form is $a_n = (n^2 + 1)2^n + 1$.
2. A 3rd order polynomial P has the property that $P(1) = 1$, $P(2) = 18$, $P(4) = 17$, and $P(5) = 23$. Find $P(3)$.
3. Check whether there exists a quintic P such that $P(0) = 0$, $P(1) = 1$, $P(2) = -2$, $P(3) = 3$, $P(4) = -4$, $P(5) = 5$, and $P(6) = -3$.

Problem Section 4

1. What happens if we add the solution to our recurrence back into the recurrence? As in, what if we have $a_{n+2} = a_{n+1} + a_n + F_n$, where the inhomogenizing term has the same recurrence relation as the rest of the recurrence?

2. "Verify" the formulae for sums of arithmetic and geometric series using a cool application of inhomogenous recurrence relations.

3. Find a way to get the partial sums of a recurrence relation in explicit form. This is really cool, so I highly suggest you do it.

More

For more on finite calculus, including a great tutorial, I'm again going to suggest

`http://www.stanford.edu/~dgleich/publications/finite-calculus.pdf`.

On the subject of characteristic polynomials, there's a wonderful compilation of good problems (and much of the same material as here) at

`http://mathcircle.berkeley.edu/BMC3/Bjorn1/Bjorn1.html`.

Wikipedia is, as always, your friend. Its article on recurrences is pretty good; find it at

`http://en.wikipedia.org/wiki/Recurrence_relation`.